Angle decomposition for wave equation migration

Cosmin Macesanu, Morgan Brown, Joe Higginbotham

February 10, 2010

Wave Imaging Technology Inc.

True Angle Decomposition for WEM

- Motivation: gathers are essential for
  - Velocity update
  - AVO/AVA analysis…

Angle is the preferred domain!

- Offset does not equal incidence angle!
  (particularly with lateral velocity variation)

- Measure angle *at the reflector*
  - Better AVO information
  - Better azimuthal (fracture) information
  - Better *velocity* information

Migration through downward continuation

Superposition of plane waves

$$U(t,x,z)=\sum_{\omega,k_x} U(\omega,k_x,k_z)\,e^{i(\omega t-k_x x-k_z z)}$$

the dispersion relation:
encodes the dynamics

$$k_x^2+k_z^2=\frac{\omega^2}{v^2}$$

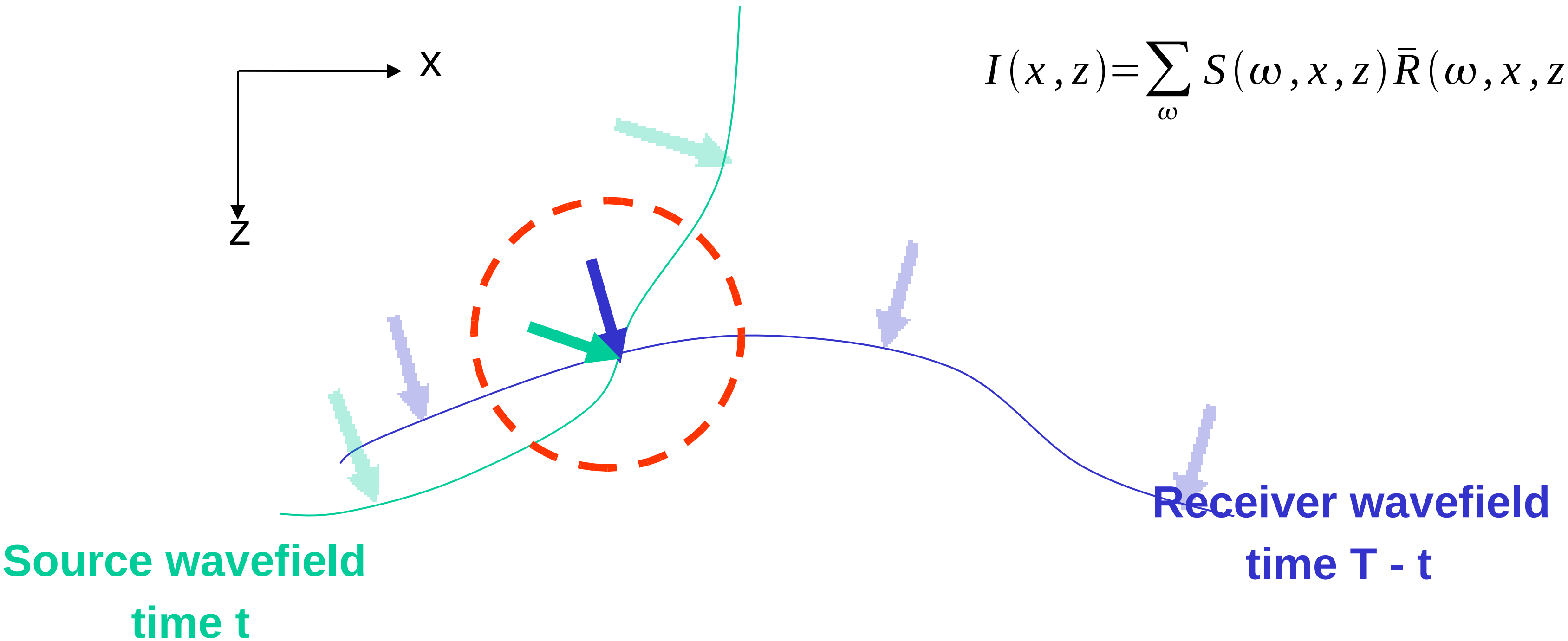
Hence

$$U(\omega,k_x,z+dz)=U(\omega,k_x,z)\,e^{-ik_z dz}$$

Imaging condition

$$I(x,z) = \int_{t=0}^{T} S(t,x,z)\, R(T-t,x,z)\, dt$$

$$I(x,z) = \sum_{\omega} S(\omega,x,z)\, \bar{R}(\omega,x,z)$$



- We work in the ω -(k,x) domain

Standard angle gather creation (Sava and Fomel, 2003)

- based on subsurface offset gathers and slant stack
  - Extra dimensionality - offset h

$$I(x,z,h)=\sum_{\omega} S(\omega,x+h,z)\bar{R}(\omega,x-h,z)$$

  - slant stack applied after shots stack

$$I(x,z,\alpha)=\sum_{h} I(x,z+h\tan(\alpha),h)$$

  - poor resolution in angle domain

Expensive, particularly in 3D; subsampling in x,y may be needed

Angle computation based on P waves

- For each image point (x,y,z), compute *propagation direction vectors* for source and receiver wavefields

direction vector ~ gradient of the scalar wavefield

$$\vec{P}(t,x,y,z) = \nabla U(t,x,y,z)$$

In wavenumber space, derivative -> k

$$\vec{P}(\omega,x,y;z) = \sum_{k_x,k_y} i\vec{k}\,U(\omega,k_x,k_y;z)\,e^{-i(k_x x + k_y y)}$$

$$k_z = \sqrt{\frac{\omega^2}{v^2} - k_x^2 - k_y^2}$$

We need the P vector at a particular time:

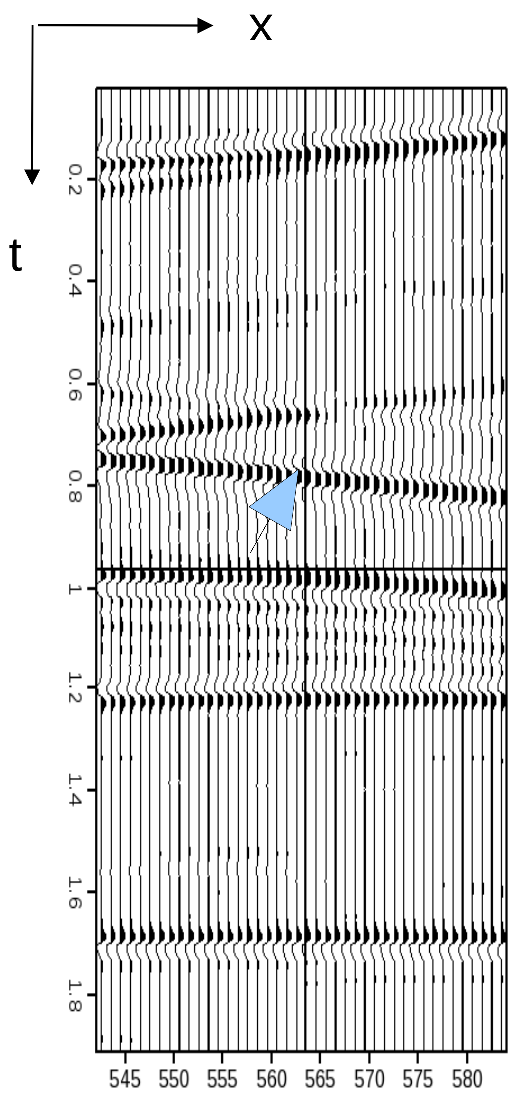
$$\vec{P}(t,x,y,z)=\sum_{\omega}\vec{P}(\omega,x,y,z)e^{i\omega t} \quad ; \qquad t=\tau$$

Use the fact that the source wave S approximates a delta function (*) in time domain, and extract required P vector through convolution

$$S(x,y,z,t) \simeq \delta(t-\tau) \quad ; \quad \tau=\tau(x,y,z)\leftarrow traveltime$$

$$\vec{P}(\tau)=\int_t \vec{P}(t)S(t)\,dt=\sum_{\omega}\vec{P}(\omega)S(\omega)$$

(*) approximation valid for downward continued waves



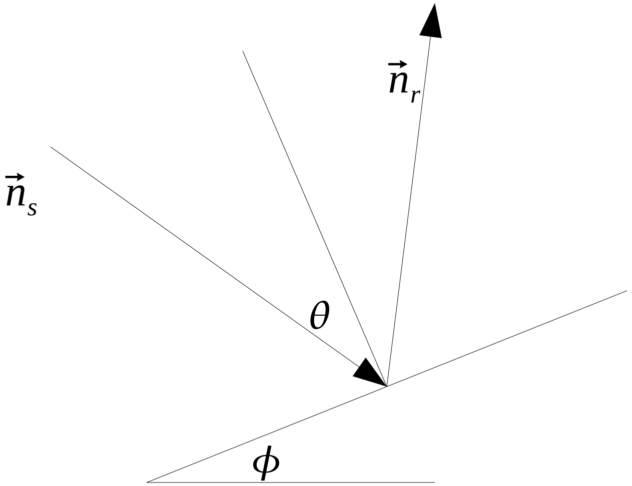
Receiver waves

Angle reconstruction from p-vectors

Compute averaged, normalized p-vectors

$$\langle \vec{P}(x,y) \rangle_{t=\tau} = \frac{\langle \sum_{\omega} \vec{P}(\omega) \bar{S}(\omega) \rangle_{x,y}}{\langle \sum_{\omega} S(\omega) \bar{S}(\omega) \rangle_{x,y}}$$

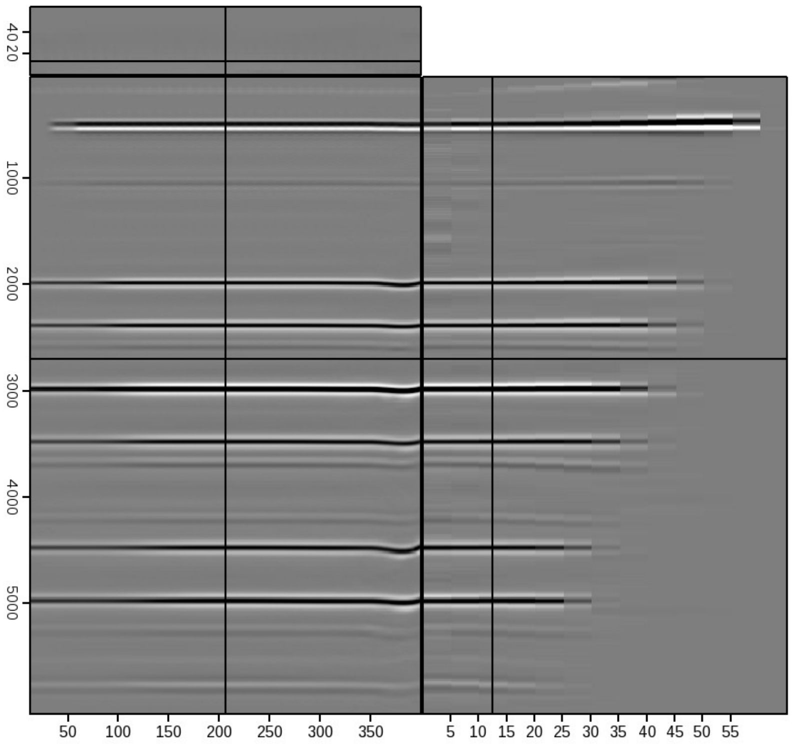
$$\vec{n}_S = \frac{\vec{P}_S}{\sqrt{\vec{P}_S^2}} \quad ; \quad \vec{n}_R = \frac{\vec{P}_R}{\sqrt{\vec{P}_R^2}}$$

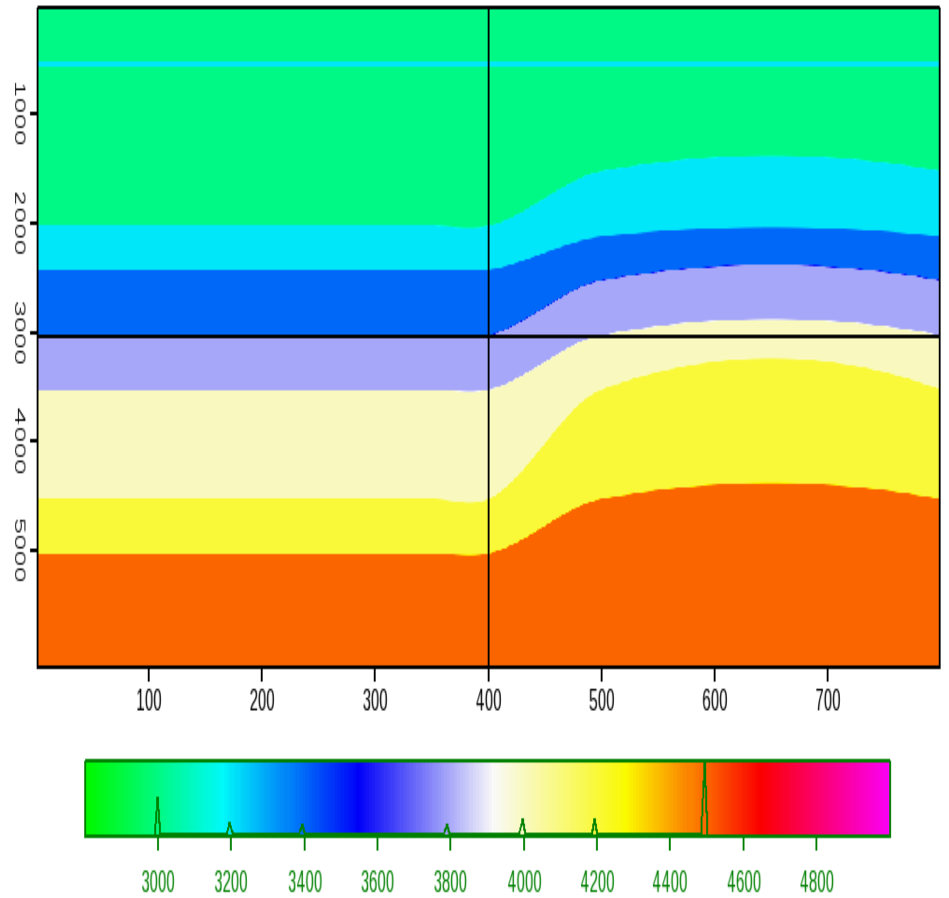


- Incidence angle, dip angle, and azimuth angle from two vectors
- Define angle “bins”, put image energy at (x,y,z) into correct bin

Angles are computed at each image point

Angle gathers for a layered 2d synthetic model





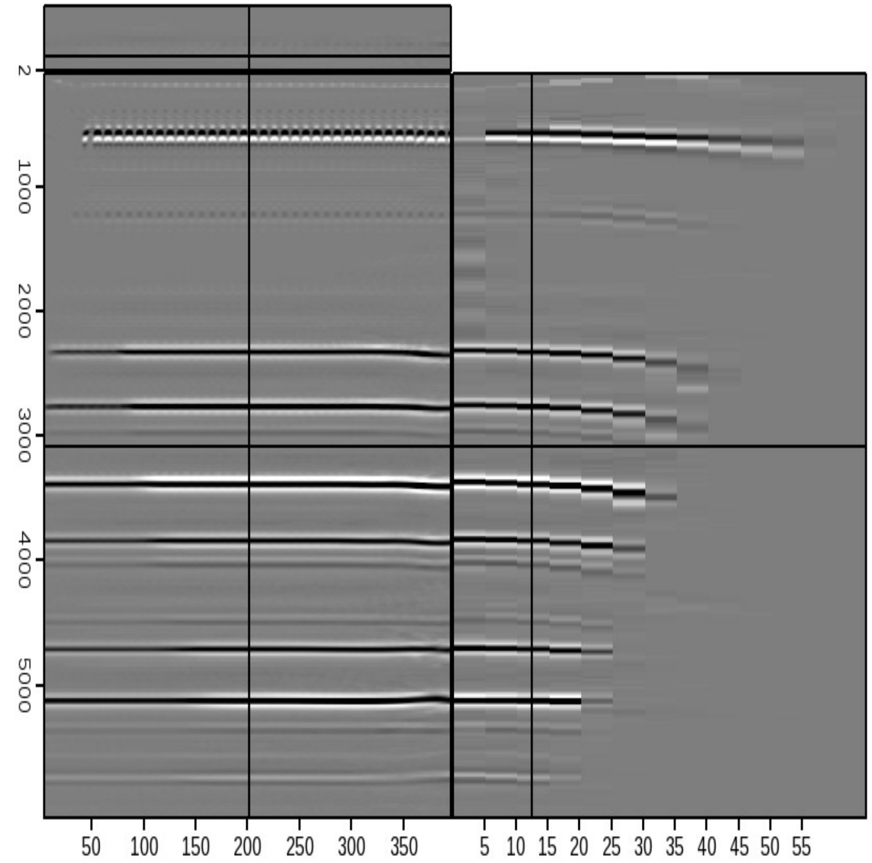
Real events are flat; multiples show curvature

Velocity update

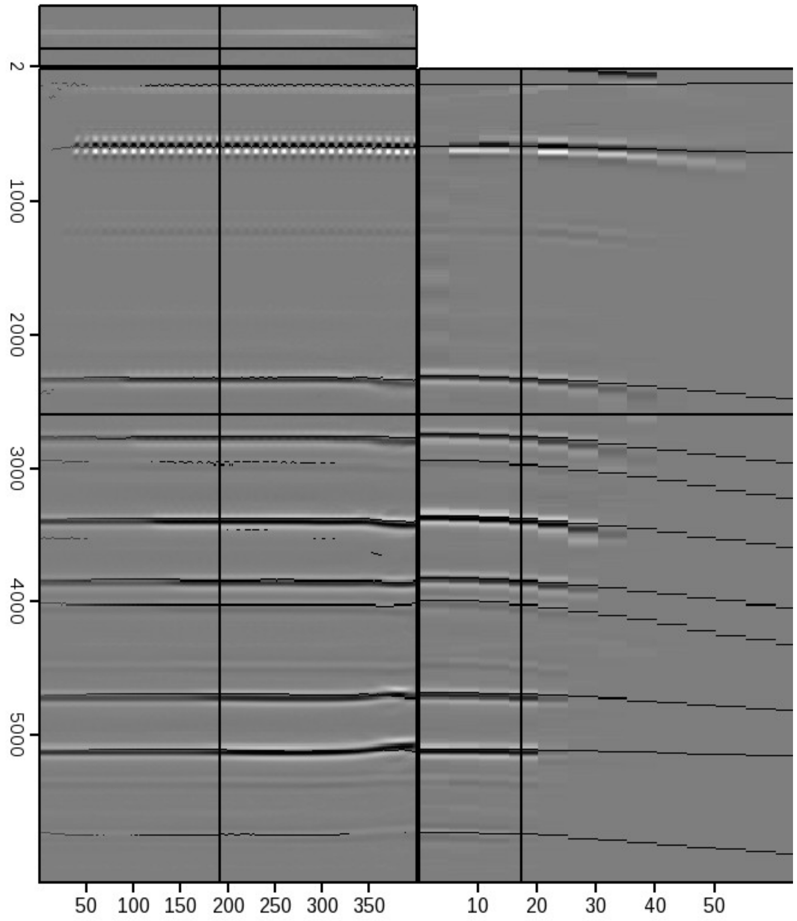
Migration with the wrong velocity:
v = 3500 m/s
results in residual parabolic moveout:

$$\Delta z(\theta) = z_0 c_p sin^2(\theta)$$

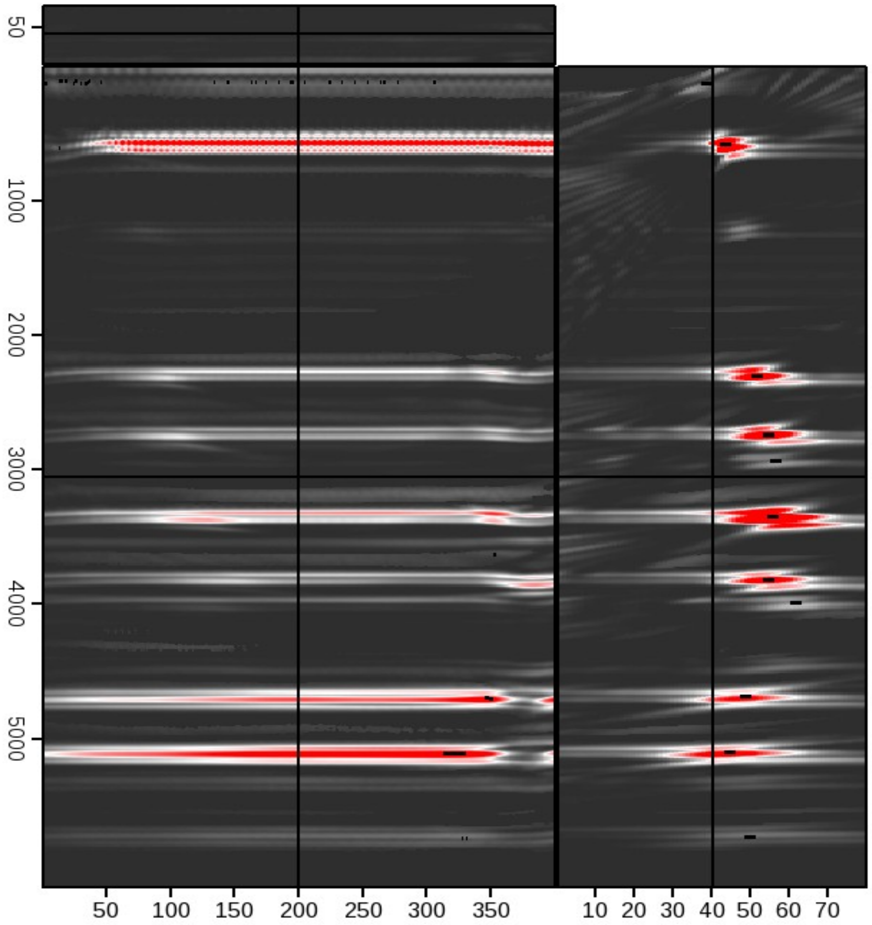
- Curving up: velocity too slow
- Curving down: too fast



Velocity picking



Parabola moveout fits



Coherence panels

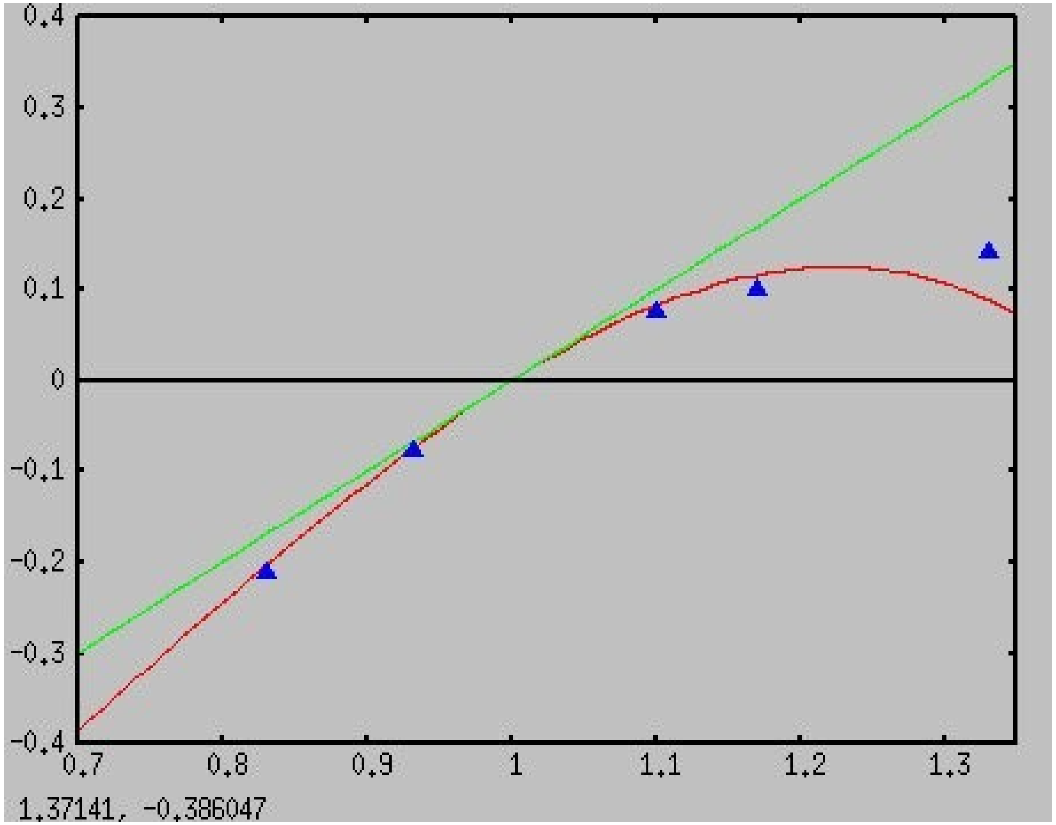
Residual moveout – parabolic coefficient

green: linearized approx

$$c_p = \rho - 1 \quad ; \quad \rho = \frac{v}{v_0}$$

red : 4th order

$$c_p = (\rho^2 - 1)\frac{2 - \rho^2}{2}$$



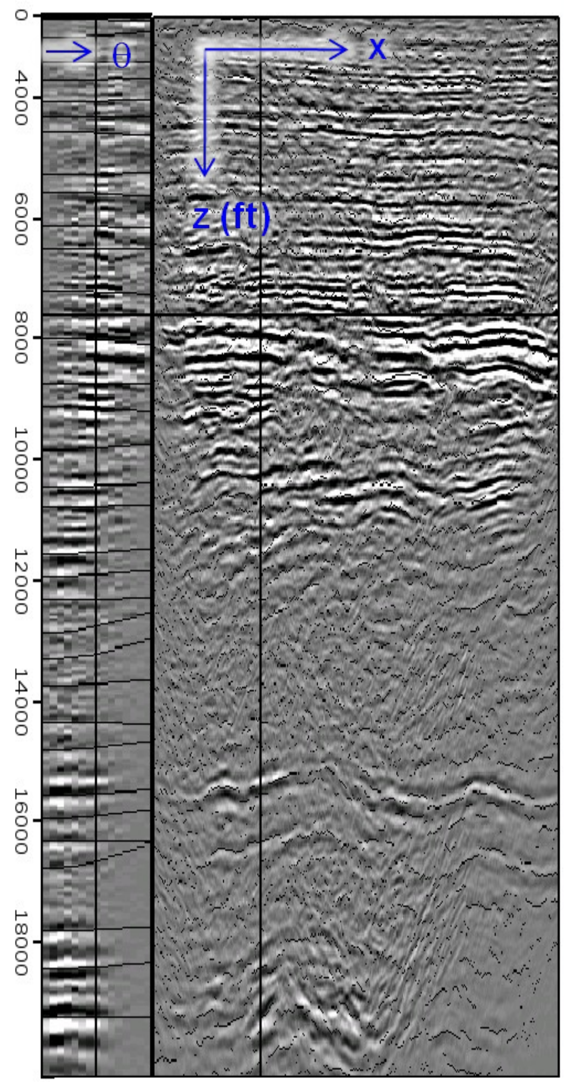
Test measured values of c_p (which rely on curvature of angle gathers) versus theory predicted values;

good agreement, once higher accuracy formula is used

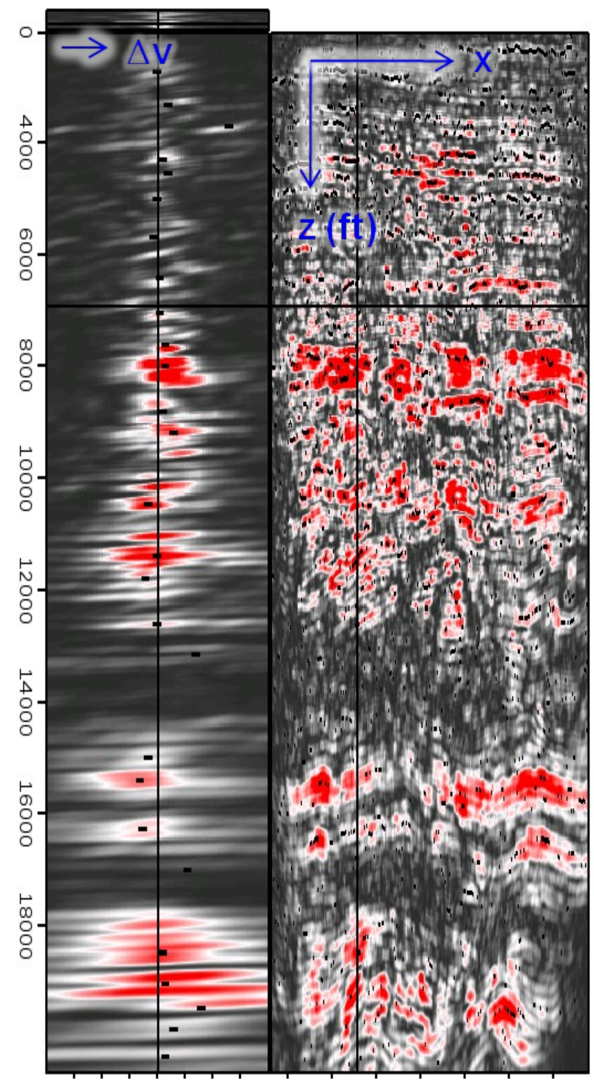
Field data example

Incidence angle gathers

- Shallow: illustrates acquisition deficiencies
  - Poor shallow image
  - Noisy images
- Velocity estimation:
  - Automatic picking of large angle gather volumes
  - Update velocity at every image point



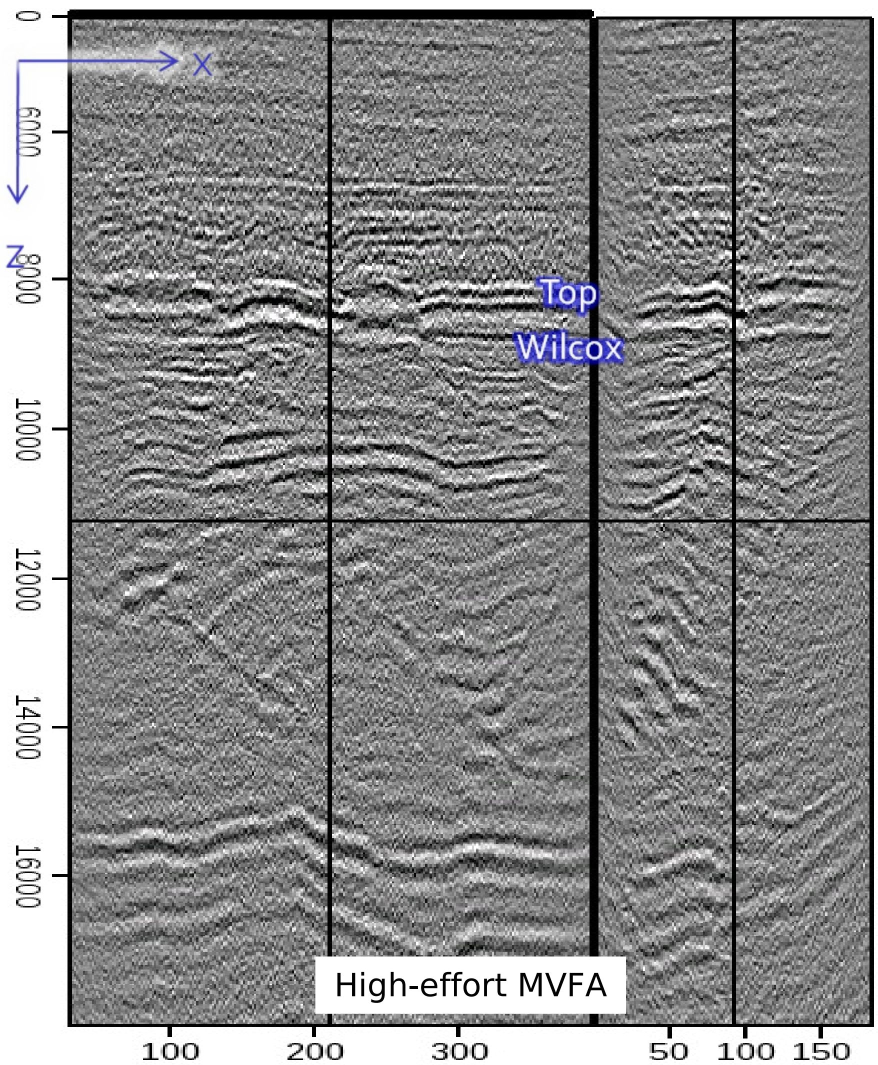
Angle gather target line

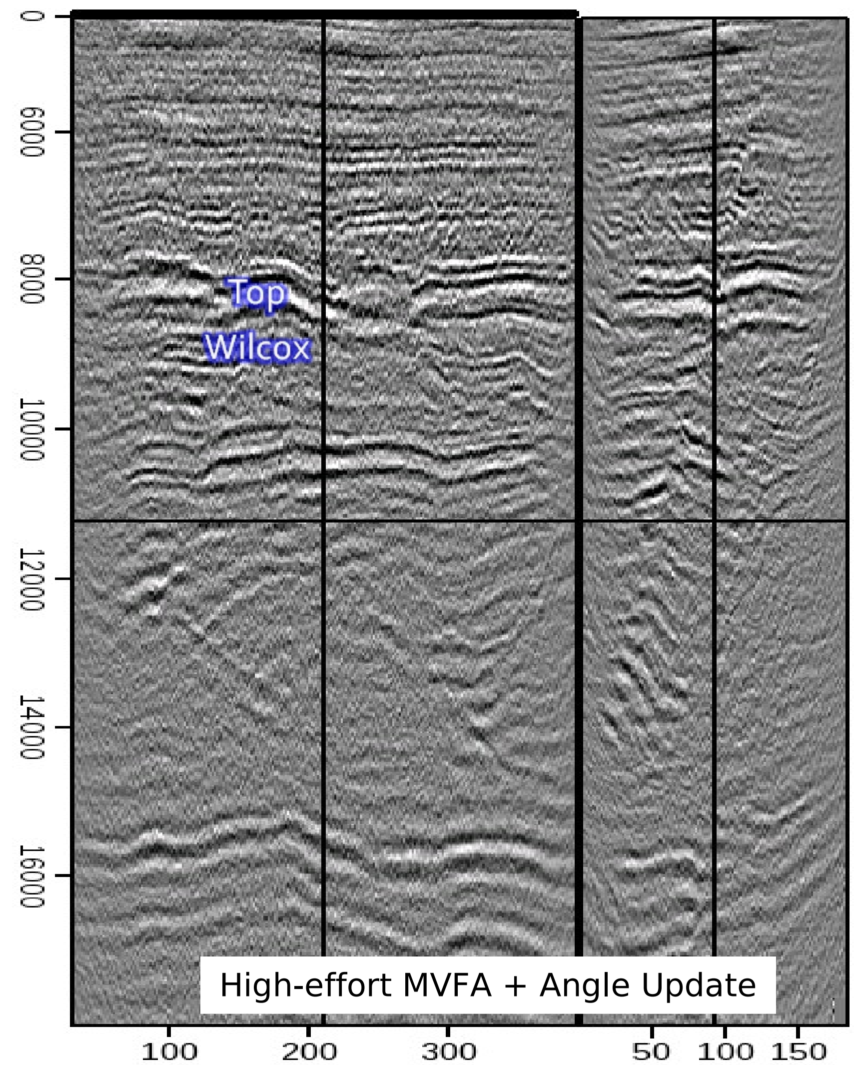


Residual Velocity panel

Data courtesy ECHO Geophysical

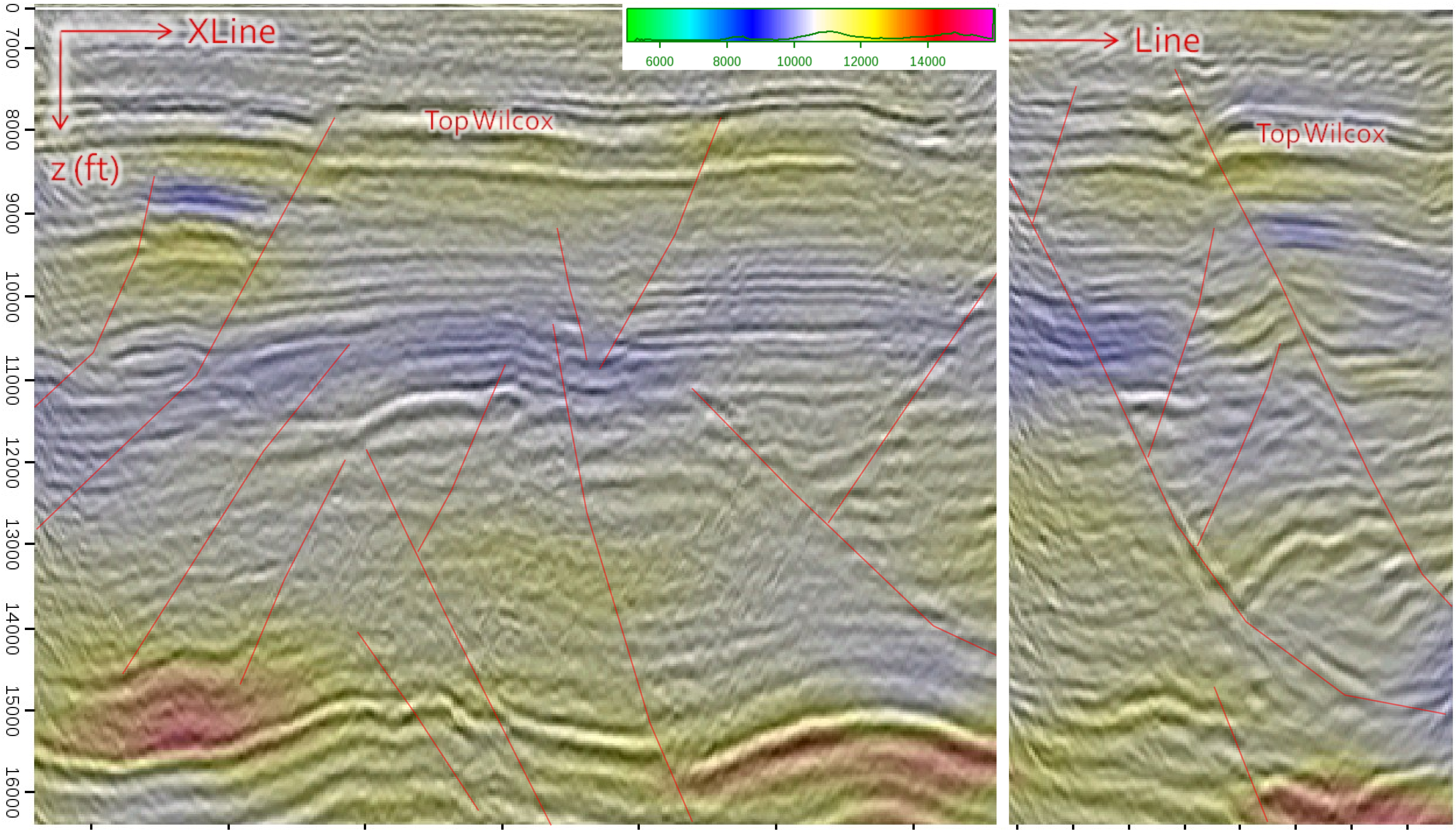
Angle gathers update





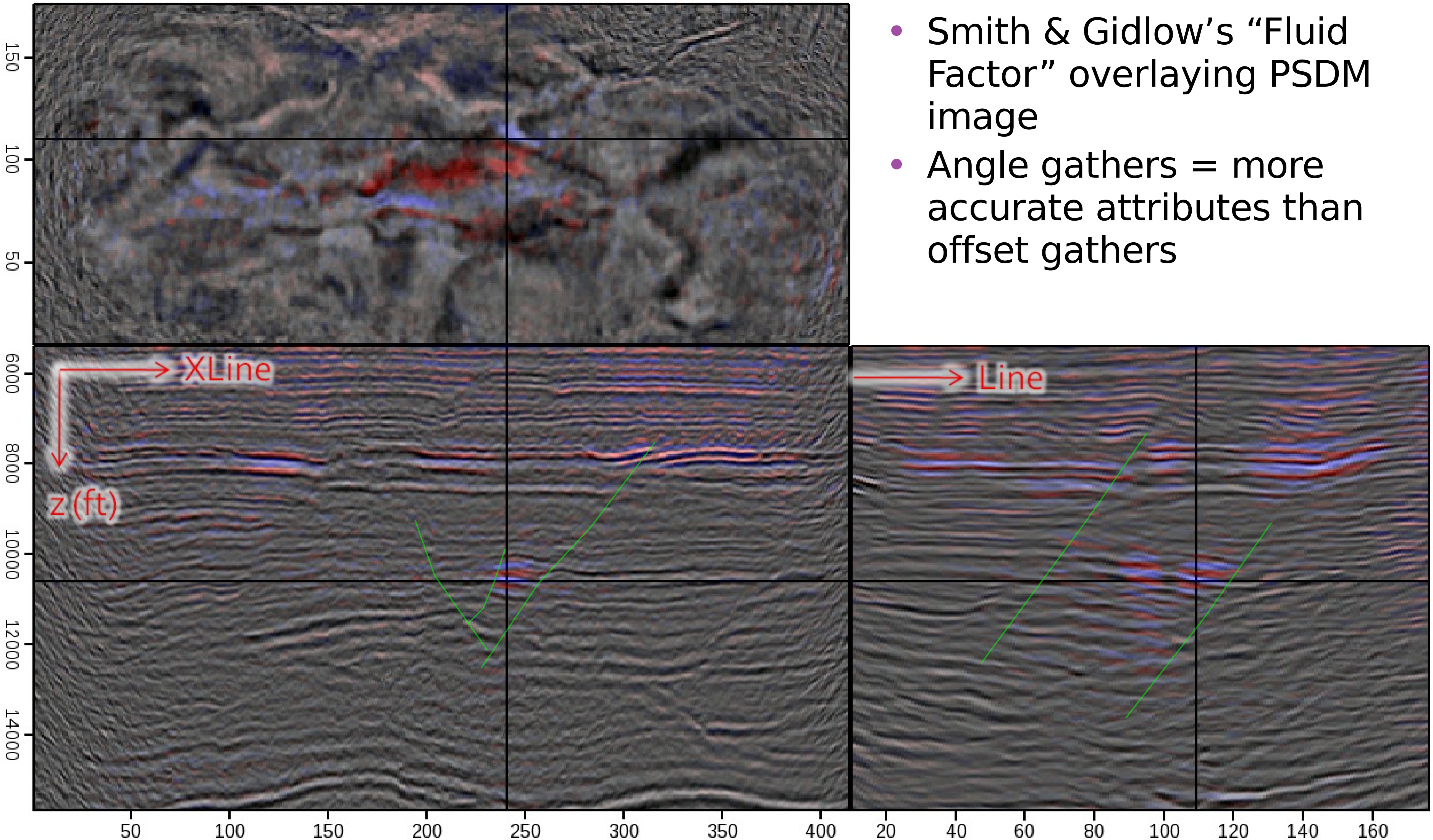
Data courtesy ECHO Geophysical

Velocity Model has Interpretive Value



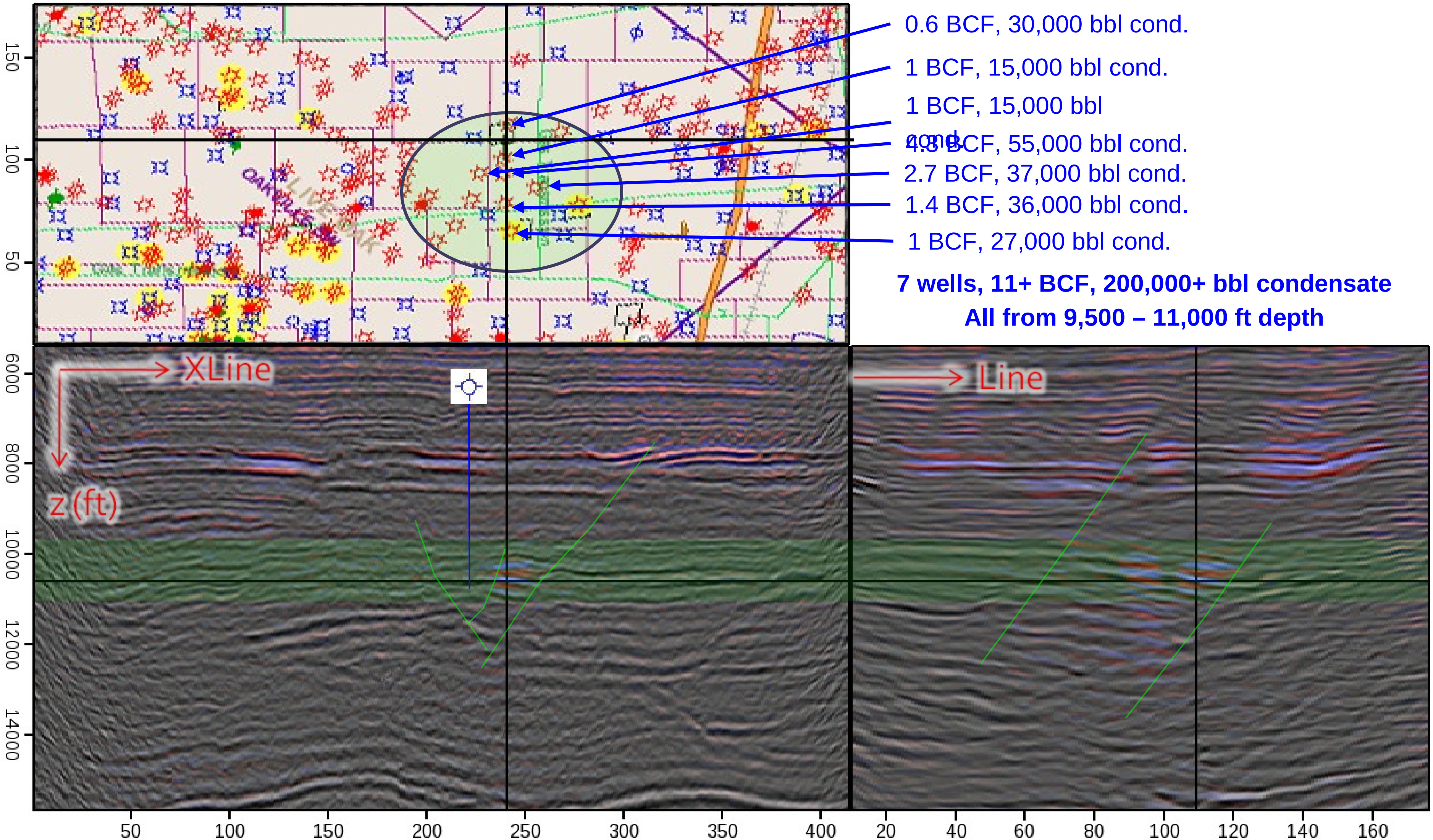
Data courtesy ECHO Geophysical

Attributes from Angle Gathers



- Smith & Gidlow's "Fluid Factor" overlaying PSDM image
- Angle gathers = more accurate attributes than offset gathers

Attributes from Angle Gathers



Conclusions

- We present a new method of building common image point angle gathers with WEM
- Less expensive in disk space, time, than current approaches.
- Based on local wavefield direction --> more accurate than methods relying on offset-to-angle conversion through raytracing
- Allows velocity update using WEM generated gathers

Appendix – Traveltime computation in the ω domain

Test the assumption of the source wave as a delta function

$$\tau \sim \int_t t\, S(t)\bar{S}(t)\,dt \sim \int_\omega \left[\frac{d}{d\omega}S(\omega)\right]\bar{S}(\omega)\,d\omega$$

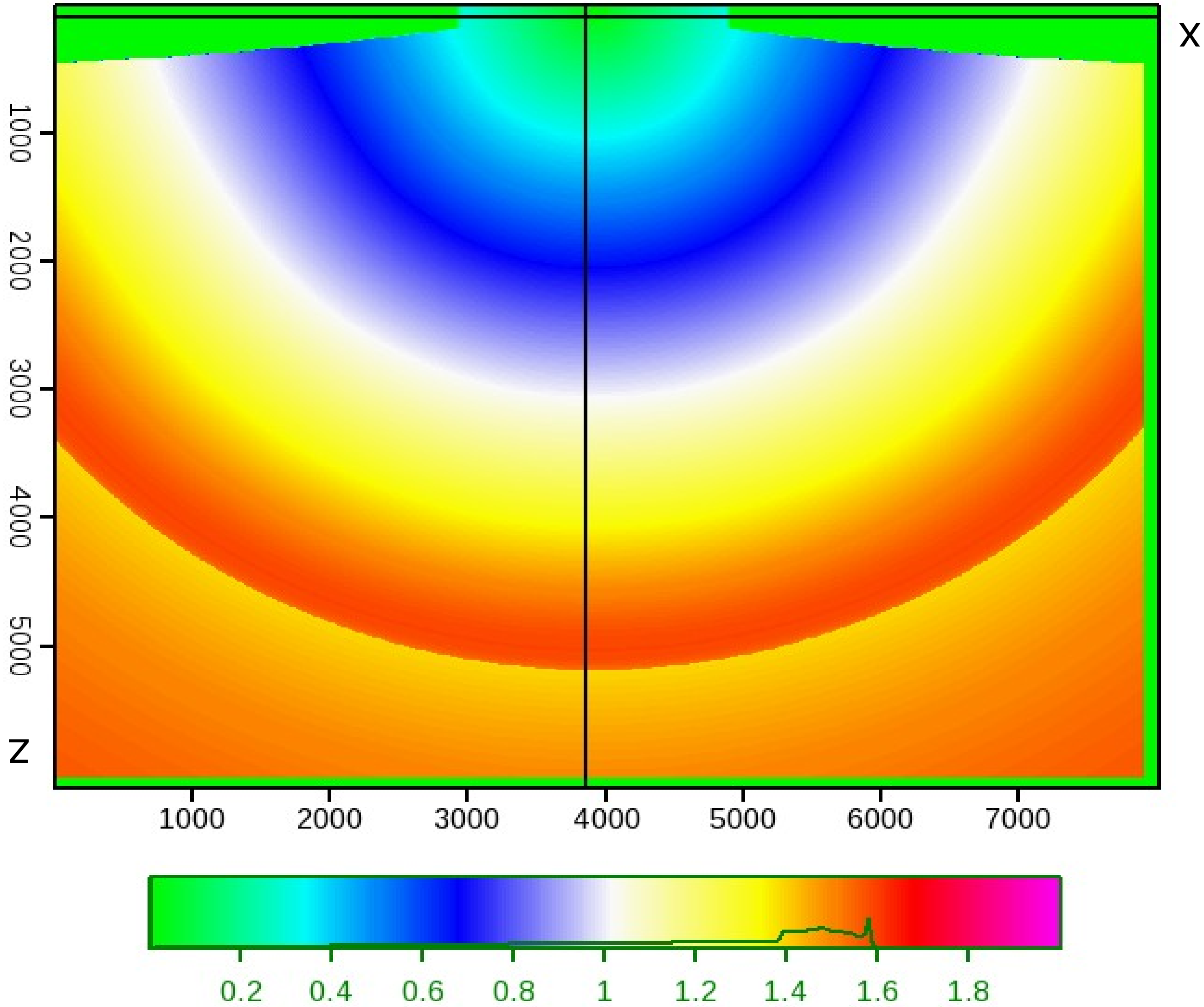
$$\sum \frac{S(\omega+\Delta\omega)-S(\omega-\Delta\omega)}{2i\Delta\omega}\bar{S}(\omega) = \int_t S(t)^2 \frac{e^{i\Delta\omega t}-e^{-i\Delta\omega t}}{2i\Delta\omega}$$

$$\simeq |S^2| \;\tau \left(1-\frac{(\tau\Delta\omega)^2}{6}+\ldots\right)$$

$$\textit{expansion parameter} \;:\; \tau\Delta\omega \sim \frac{2\pi\tau}{T}$$

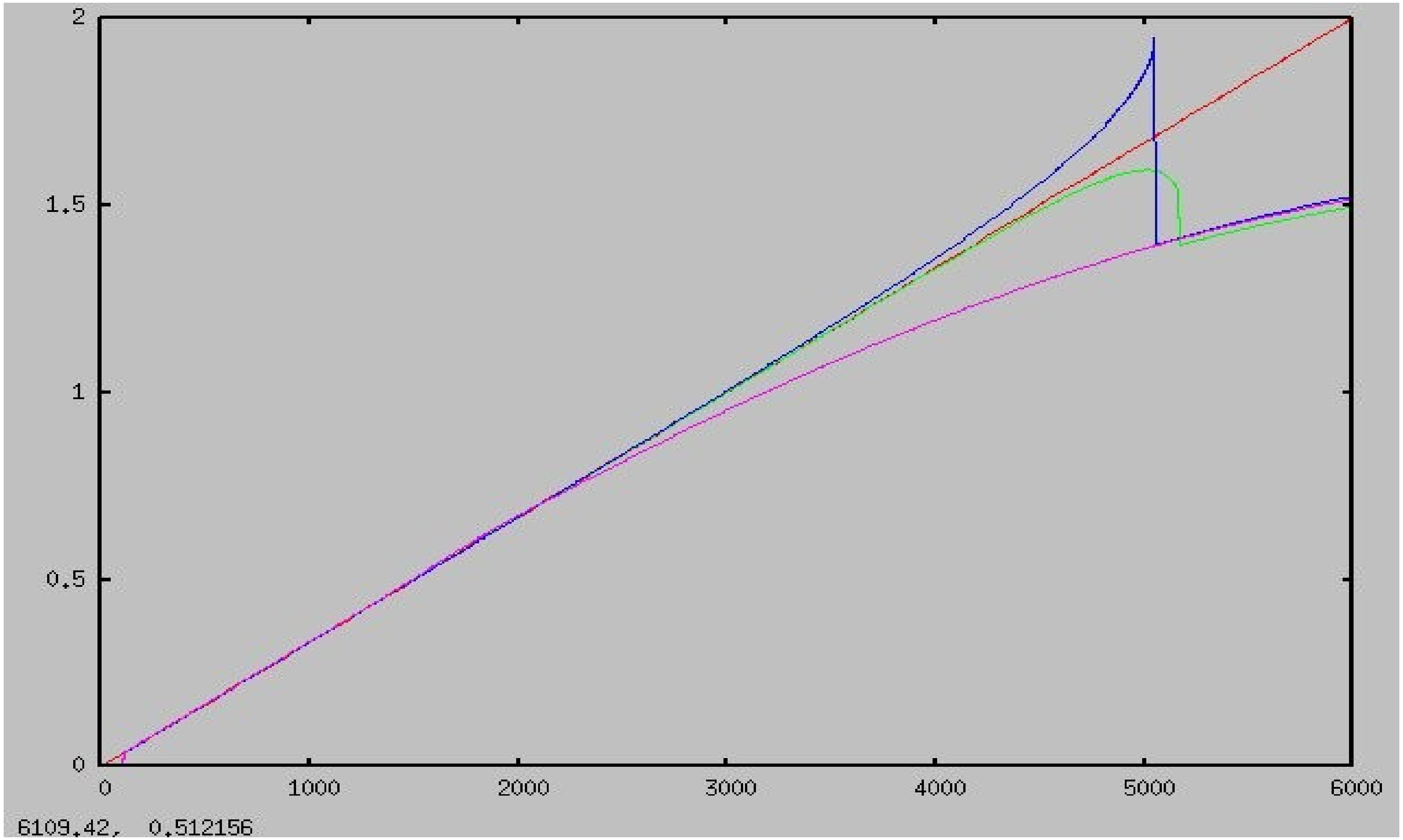
For accurate results, need tau << T/2
(values >T/2 map onto negative t-T domain)

Traveltime in constant velocity medium



V=3000 m/s

Traveltime accuracy



Traveltime as a function of z; using higher order approximations (T = 5.5 s)