Wave Equation Migration Velocity Focusing Analysis

Joseph H. Higginbotham and Morgan P. Brown, Wave Imaging Technology Inc.*
Robert G. Clapp, Stanford University

Summary

We present the theory behind a velocity estimation technique which uses the time-shift imaging condition for wave equation prestack depth migration. The migration time-shift parameter is converted to a perturbation in RMS velocity, which can be converted to interval velocity. Our approach can resolve velocity errors in the presence of subsurface complexity that would hamper velocity analysis with surface data. Moreover, it has no dip or offset limitations.

Introduction

It is widely understood that in regions with significant geologic complexity, velocity analysis using depth migration (MVA) is superior to methods which operate on prestack data. Migration in general, and depth migration in particular, simplify prestack data by correcting the effects of offset, reflector dip, and propagation from source to receiver in a heterogeneous medium. When the migration velocity is incorrect, migration will incorrectly position the surface data in depth. The most common MVA technique attempts to flatten Kirchhoff common-offset depth migration gathers by measuring depth error as a function of offset and perturbing the migration velocity accordingly.

Kirchhoff offset gathers are known to exhibit artifacts in complex examples, which are not seen in wave equation depth-migrated images (Stolk and Symes, 2004). Claerbout's (1985) zero-time/zero-offset prestack imaging condition forms the basis of most wave equation MVA techniques. Subsurface offset gathers can be converted to angle gathers by slant-stacking (Prucha et al, 1999; Sava and Fomel, 2003), and have been used for MVA (Clapp, 2001; Sava, 2004). However, the slant stack may itself introduce spurious artifacts. Shen et al. (2003) describe a velocity update method which uses mis-focusing in subsurface offset directly.

Another class of MVA methods uses the other wave equation prestack focusing criterion—mis-focusing in time—to quantify velocity errors. MacKay and Abma (1992) construct "time-shift" gathers by phase-shifting source and receiver wavefields in shot record migration. While time-shift gathers can be converted to angle gathers (Sava and Fomel, 2006), MacKay and Abma measured the depth corresponding to best focusing and invoked a relationship attributed to Faye and Jeannot (1986) to relate the measured depth error to a velocity perturbation. The technique has small reflector dip and small offset assumptions. Audebert and Diet (1992) outline an approach to partially overcome these limitations.

We take an alternate view of migrated time-shift gathers, similar to Higginbotham (1993). Rather than pick the focusing depth, we convert the time-shift parameter directly to a perturbation in RMS velocity, using a relationship that has no dip or offset limitations, unlike that used by Higginbotham (1993). The RMS velocity can be converted to interval velocity, using, for instance, the Dix equation. We call our method Wave Equation Migration Velocity Focusing Analysis, or MVFA for short.

In this abstract we outline the theory behind the MVFA method, proving some key results. A companion paper (Brown et al., 2008) illustrates MVFA applied to a complex synthetic land example.

Theory

Consider the moveout equation (Levin, 1971) for constant velocity c for a dipping reflector with dip angle θ, half offset h, zero offset time t_0, and arrival time t:

$$c^2 t^2 = c^2 t_0^2 + 4h^2 \cos^2 \theta, \qquad (1)$$

This equation is often applied to a common midpoint (CMP) gather in which the midpoint is the same for all traces. For a constant velocity medium this equation parameterizes the travel time in terms of h and t_0 with a single parameter called the stacking velocity which is equal to c cos(θ) . This single parameter can be used to align the reflection information, from the same planar reflector but not in general from the same reflection point, that is recorded on traces of different offset. Here the equation will not be used in this way. Instead the reflection point will be held constant and the equation will be used to relate depth imaging errors derived from time shift gathers to errors in the imaging velocity. Since moveout is not the objective here and since downward continued "midpoints" may differ for rays arriving at and reflecting from the reflection point, the equation will be referred to as "the travel time equation".

In the course of shot record migration the surface data and the simulated shot are both downward continued to some depth Z using the current best velocity information for the overburden above this depth. This velocity may change arbitrarily. At the end of this derivation the time t will be interpreted as the travel time through a "replacement overburden" having an interval velocity with characteristics that allow use of the travel time equation from the surface down to the downward continued depth. Changes in the time at this depth, $\Delta t = t' - t$, will be interpreted as the time shifts associated with building time shift gathers and will be given the new name, τ. The intersection of the reflection surface with the vertical line through the surface location at which the time shift gathers are computed defines the fixed analysis point. The depth of the reflector will be related to t_0 exactly for the constant velocity case even for dipping events. When the velocity is represented as a RMS velocity then it can be shown that the relation of t_0 to depth is exact for flat events and approximate for dipping events. Since the half offset h, the zero offset time t_0, and even the exact velocity, are not easily available during shot record downward continuation, the goal will be to eliminate these variables form the equation in favor of known quantities such as the migration velocity along with t and τ as defined above.

Define Z as the vertical depth to the reflection point when h=0. Then we can rewrite equation (1) in terms of Z:

$$c^2 t^2 = \frac{4Z^2}{\cos^2\theta} + 4h^2\cos^2\theta, \qquad (2)$$

Holding t, θ, and h constant, we can perturb the velocity in equation (2), which leads to a perturbed depth. Call that perturbed velocity v and the perturbed depth Z' and rewrite equation (2):

$$v^2 t^2 = \frac{4Z'^2}{\cos^2\theta} + 4h^2\cos^2\theta, \qquad (3)$$

Subtracting equation (2) from (3) allows us to write a relationship between a perturbation in velocity and a perturbation in depth:

$$\Delta v = \frac{\overline{Z}}{\overline{v}} \frac{4\Delta z}{t^2\cos^2\theta}, \qquad (4)$$

where

$$\Delta v \equiv v - c; \quad \Delta Z \equiv Z' - Z; \quad \frac{\overline{Z}}{\overline{v}} = \frac{Z + \Delta Z/2}{c + \Delta v/2} \qquad (5)$$

Returning to the traveltime relation (2), we can evaluate the equation at h=0, then relate ΔZ to a perturbation in zero offset traveltime, Δt_0:

$$\Delta Z = \Delta t_0 \frac{c}{2}\cos\theta. \qquad (6)$$

We can also manipulate equation (2) to relate perturbations in zero offset traveltime, Δt_0, to perturbations in non-zero offset traveltime Δt. Perturbing t_0 and t, then doing some algebra and rearranging, we obtain:

$$2t\Delta t\left(1 + \frac{\Delta t}{2t}\right) = 2t_0\Delta t_0\left(1 + \frac{\Delta t_0}{2t_0}\right). \qquad (7)$$

Now, tying this analysis back to time-shift imaging, we are ready to begin recasting the equation for velocity perturbation (4) in terms of the τ from the time-shift imaging condition in the migration. Starting with equation (4), we insert the relation for average depth-to-velocity ratio from equation (5) and then use equation (6) to replace the ΔZ terms with Δt_0 terms. Simplifying, we obtain:

$$\Delta v = \frac{ct_0\Delta t_0}{t^2}\left(1 + \frac{\Delta t_0}{2t_0}\right)\left(1 + \frac{\Delta v}{2c}\right)^{-1}. \qquad (8)$$

Now let us modify equation (8) further. Recognizing a term that looks like the right-hand side of equation (7) we substitute and obtain:

$$\Delta v = \frac{c\Delta t}{t}\left(1 + \frac{\Delta t}{2t}\right)\left(1 + \frac{\Delta v}{2c}\right)^{-1}. \qquad (9)$$

The last component of equation (8) is further modified by applying algebraic manipulations:

$$\left(1 + \frac{\Delta v}{2c}\right)^{-1} = \left(1 - \frac{\Delta v}{v}\right)\left(1 - \frac{\Delta v}{2v}\right)^{-1}. \qquad (10)$$

Using the fact that

$$c = v\left(1 - \frac{\Delta v}{v}\right), \qquad (11)$$

we can use relations (10) and (11) to modify equation (9) into the following expression for Δv:

$$\Delta v = \frac{v\Delta t}{t}\left(1 - \frac{\Delta v}{v}\right)^2\left(1 - \frac{\Delta v}{2v}\right)^{-1}\left(1 + \frac{\Delta t}{2t}\right). \qquad (12)$$

The final step is to associate the Δt's in equation (12) with τ's from migration:

$$\Delta v = \frac{v\tau}{t}\left(1-\frac{\Delta v}{v}\right)^2\left(1-\frac{\Delta v}{2v}\right)^{-1}\left(1+\frac{\tau}{2t}\right). \quad (13)$$

We can simplify equation (13) to obtain a more pleasing formula. Defining for simplicity the relative velocity and time shifts α and β:

$$\alpha \equiv \frac{\Delta v}{v}; \quad \beta \equiv \frac{\tau}{t} \quad (14)$$

we can recast equation (13) as a quadratic equation in α and β:

$$\frac{\beta^2}{2} + \beta - \frac{\alpha\left(1-\frac{\alpha}{2}\right)}{\left(1-\alpha\right)^2} = 0. \quad (15)$$

By solving equation (15), expanding the square root to three terms, and keeping only terms of order α^2, we obtain an approximation to equation (13):

$$\frac{\tau}{t} = \frac{\Delta v}{v}\left(1+\frac{\Delta v}{v}\right). \quad (16)$$

Equation (16) is quite accurate. For a relative velocity perturbation of 10% (α=0.1), the correct result is β=0.1111…, while equation (16) yields a result of β=0.11, implying an error of only 0.1%.

An important conclusion of the derivation above is that the relationship between migration time shift and velocity perturbation does not depend on reflector dip or offset. The traveltime equation (1) does handle dip, but the dip angle drops out of the calculation. The depth focusing analysis relations used by MacKay and Abma (1992) has both a small dip and small offset requirement.

The derivation above is fairly straightforward to repeat for depth-variable velocity. The moveout equation (1) is well-defined and accurate for dipping layers in RMS velocity. The first stumbling block is reached when we attempt to relate reflector depth to zero-offset traveltime, t_0. Ray bending makes the analysis ill-posed. One possibility is to assume that the reflector depth is controlled by the average velocity. After making that assumption, the above derivation may be repeated, and a result similar to equations (13) and (16) is obtained, except that v is replaced with RMS velocity.

Implementation

Velocity analysis using the MVFA concept begins with so-called "time-shift gathers", computed in our case with a wave equation shot record depth migration algorithm. The reader is referred to Sava and Fomel (2006) for a detailed discussion on the time-shift imaging condition, along with many data examples. Figure 1 illustrates an actual time-shift gather taken from a synthetic dataset migrated with an incorrect velocity.

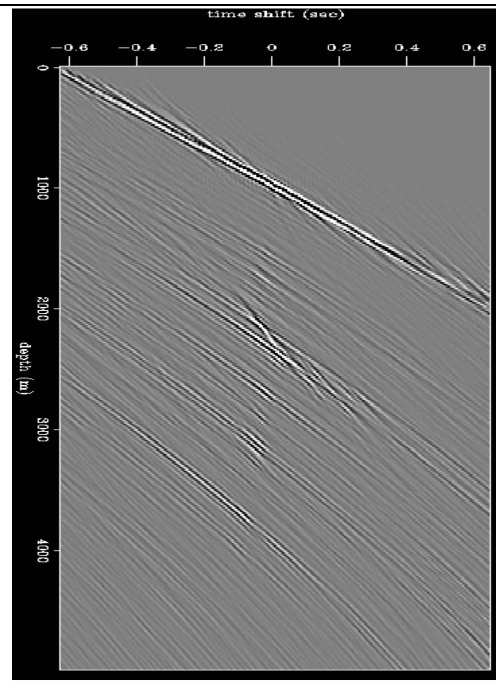

Figure 1: Time-shift gather when migration velocity was too slow below 2500 m depth. The vertical axis is depth. The horizontal axis is time-shift, τ, where zero time shift (corresponding to the conventional zero-time/zero-offset imaging condition) is in the center of the figure. The shallow velocity was correct. The energy peaks are shifted away from τ=0, by as much as about -0.15 sec.

Equations (13) and (16) represent a horizontal stretch of the time-shift axis of a time-shift gather into units of velocity perturbation, Δv. When the migration velocity is added to the velocity perturbations, the peak energy should represent a measure of the true (RMS) velocity. The depth axis of the gathers is converted to time, and a semblance operator is applied to highlight the energy peaks. In order to compute this semblance gather, the shot profile migration code must track the square of amplitudes from each contributing shot, in addition to the usual amplitude. The result is called an "MVFA gather".

Figure 2 shows MVFA gathers taken from a synthetic land dataset which is processed in detail in the companion paper (Brown et al., 2008). The top gather was computed using an incorrect migration velocity—the semblance peaks do not overlay the migration velocity, implying that the migration velocity should be slowed down or sped up. The bottom gather was computed using the correct velocity, and it can be seen that the semblance panels overlay the migration velocity, implying no speedup is needed.

To update the migration velocity, the velocity peaks can be picked and converted to an interval velocity, normally in vertical update mode.

Conclusions and Discussion

In this abstract we presented the theoretical foundation of a velocity analysis method which uses the time-shift imaging condition for prestack wave equation shot record migration. We derived equations which convert the time shift parameter from the migration to a perturbation in velocity. The equations were shown to be dip- and offset-independent in the constant velocity case.

Although the equations were derived using time-domain moveout equations, it is important to note that the time shifts themselves are measured via wave equation migration, and thus can cleanly show velocity errors under a complex overburden. In fact, if the overburden velocity is correct, then the concept of replacement velocity can unambiguously represent the overburden velocity for the purposes of MVFA. In such a manner, the relatively simple vertical update implied here could likely measure and represent a velocity perturbation beneath a complex salt, provided that the overburden velocity is accurate.

The general concept of using wave equation time-shift gathers has a natural and powerful applicability to reflection tomography applications, if an angle-dependent time-shift gather can be constructed. Perturbations in velocity (slowness, in fact) are most naturally represented by perturbations in time, which a time-shift gather measures.

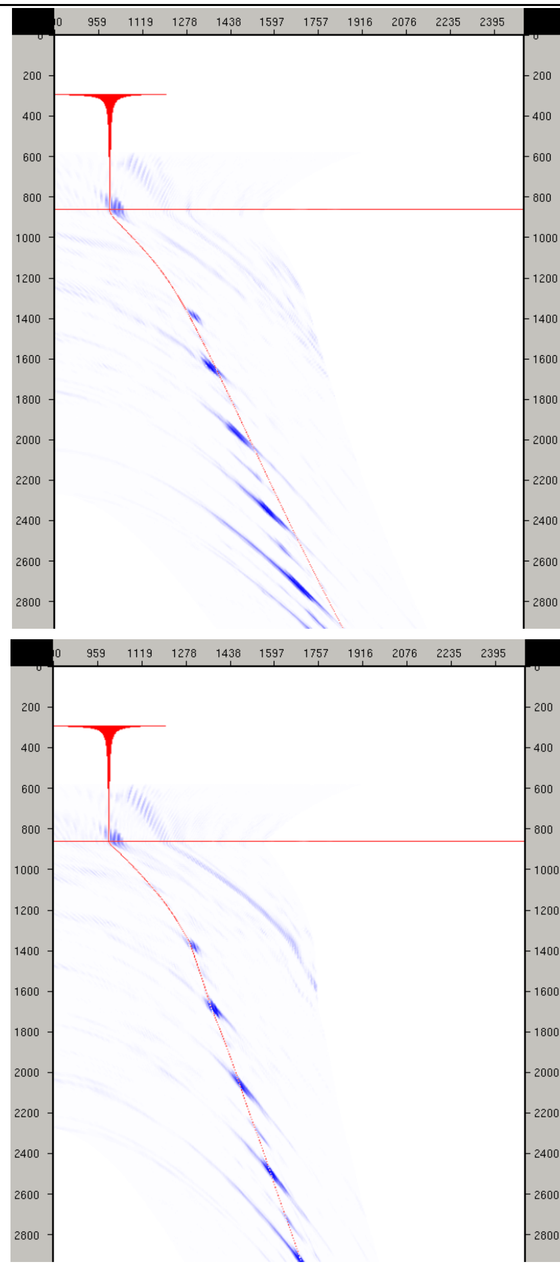

Figure 2: MVFA gathers taken from a synthetic land dataset, also shown in Brown et al. (2008). Vertical axis is time, horizontal axis is RMS velocity (800-2500 m/s). The blue peaks represent velocity implied by analysis. The red line indicates the migration velocity used. The onset of the red curve at around 300 msec represents the topographic datum. Top: After migration with initial velocity. Bottom: After migration with known velocity.