Velocity model building with Wave Equation Migration Velocity Focusing Analysis

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Summary

We describe a velocity estimation technique which uses the time-shift imaging condition for wave equation prestack depth migration. The time-shift parameter is converted to a perturbation in RMS velocity, which we convert to interval velocity. Our approach can resolve velocity errors in the presence of subsurface complexity that would hamper velocity analysis with surface data. We demonstrate the method on a realistic 2D synthetic land example.

Introduction

It is widely understood that in regions with significant geologic complexity, velocity analysis using depth migration (MVA) is superior to methods which operate on prestack data. Migration in general, and depth migration in particular, simplify prestack data by correcting the effects of offset, reflector dip, and propagation from source to receiver in a heterogeneous medium. When the migration velocity is incorrect, migration will incorrectly position the surface data in depth. The most common MVA technique attempts to flatten Kirchhoff common-offset depth migration gathers by measuring depth error as a function of offset and perturbing the migration velocity accordingly.

Kirchhoff offset gathers are known to exhibit artifacts in some cases, which are not seen in wave equation depth-migrated gathers (Stolk and Symes, 2004). Claerbout's (1985) prestack imaging condition, illustrated by Figure 1, forms the basis of most wave equation MVA techniques. Subsurface offset gathers can be converted to angle gathers by slant-stacking (Prucha et al, 1999; Sava and Fomel, 2003), and have been used for MVA (Clapp, 2001; Sava, 2004). Shen et al. (2003) describe a velocity update method which uses mis-focusing in subsurface offset directly.

Another class of MVA methods uses the other wave equation prestack focusing criterion—mis-focusing in time—to quantify velocity errors. MacKay and Abma (1992) construct "time-shift" gathers by phase-shifting source and receiver wavefields in shot record migration. While time-shift gathers can be converted to angle gathers (Sava and Fomel, 2006), MacKay and Abma measured the depth corresponding to best focusing and invoked a relationship attributed to Faye and Jeannot (1986) to relate the measured depth error to a velocity perturbation. The technique has small reflector dip and small offset assumptions. Audebert and Diet (1992) outline an approach to partially overcome these limitations.

We take an alternate view of migrated time-shift gathers similar to Higginbotham (1993). Rather than pick the focusing depth, we convert the time-shift parameter directly to a perturbation in RMS velocity, using a relationship that has no dip or offset limitations, unlike that used by Higginbotham (1993). The RMS velocity can be converted to interval velocity, using, for instance, the Dix equation. We call our method Wave Equation Migration Velocity Focusing Analysis, or MVFA for short. The theory underlying MVFA is outlined in a companion paper (Brown et al., 2008).

In this abstract, we demonstrate the efficacy of a velocity model building flow based on MVFA on a complex 2D synthetic dataset modeled after the San Joaquin Valley, California, USA. The synthetic example includes 200 m of topographic relief, a weathering layer of variable thickness, significant lateral velocity variation, and dips to 75 degrees. After building an initial velocity model, we run two iterations of MVFA and significantly reduce velocity errors, leading to an improved final image.

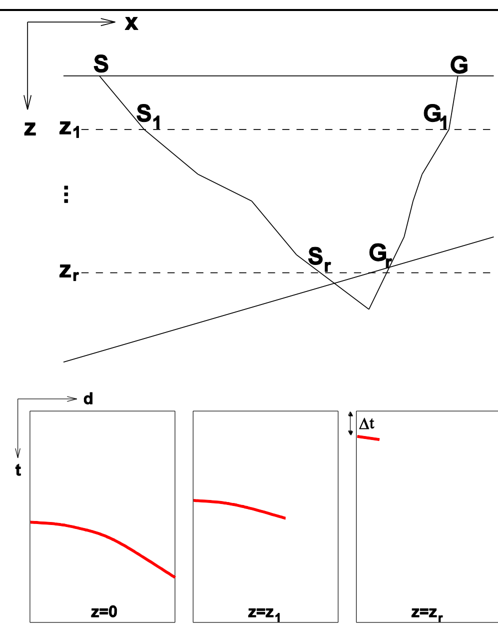

Figure 1: Illustration of Claerbout's prestack imaging condition. Top: when the velocity is wrong, the source and receiver wavefields do not focus at the true reflector depth zr. Bottom: On the downward-continued shot gather, the energy is not concentrated at zero offset and zero time, but rather shifted in time and dispersed in offset.

Wave Equation Migration Velocity Focusing Analysis

Method

While it is possible to do time-shift imaging with many types of migration algorithm, in this abstract the description and implementation is assumed to be wave equation shot record depth migration. Algorithmically, the surface data is downward continued into the earth using a one-way wave equation. A synthetic source wavelet is also downward continued, starting at the known source location. If $U_r(\mathbf{x},\omega)$ and $U_s(\mathbf{x},\omega)$ are the Fourier-transformed receiver and source wavefields at $\mathbf{x}=(x,y,z)$ and temporal frequency ω, then we form an image, $I(\mathbf{x})$, using the zero offset/zero time imaging condition:

$$I(\mathbf{x}) = \sum_{\omega} U_s(\mathbf{x},\omega)\, U_r^*(\mathbf{x},\omega). \qquad (1)$$

The asterisk (*) in equation (1) represents complex conjugation. Because the two wavefields are being multiplied point-wise at zero lag, equation (1) is equivalent to zero-offset extraction. The subsurface offset may be increased by correlating the wavefields at a non-zero spatial lag (Rickett and Sava, 2002). The summation over frequency in equation (1) produces the desired zero-time extraction.

To construct a time-shift gather, the source and receiver wavefields are both phase-shifted at a multiplicity of time-shifts, τ, to produce a gather at each (x,y) location, consistent with Sava and Fomel (2006):

$$I(\tau,\mathbf{x}) = \sum_{\omega} U_s(\mathbf{x},\omega)\, U_r^*(\mathbf{x},\omega)\, e^{2i\omega\tau}. \qquad (2)$$

Figure 2 shows a time-shift gather from a synthetic dataset. The migration velocity below 2500 m in depth was too slow, so the events are best focused at some τ away from τ=0. The first step in the MVFA method is to apply a semblance measure to a time-shift gather (MacKay and Abma used an envelope function). The depth axis of the semblance gather is then converted to time, using the migration velocity at the appropriate (x,y) location. Finally, the τ axis of the semblance gather is transformed to a perturbation in RMS velocity, Δv, according to the relation

$$\frac{\Delta v}{v}\left(1-\frac{\Delta v}{2v}\right)\left(1-\frac{\Delta v}{v}\right)=\frac{\tau}{t}\left(1+\frac{\tau}{t}\right), \qquad (3)$$

where v is the RMS migration velocity and t is the vertical axis of the time-converted gather. Equation (3) has an

$$\frac{\Delta v}{v}\left(1+\frac{\Delta v}{2v}\right)=\frac{\tau}{t}. \qquad (4)$$

accurate approximation, with an error of only 0.1% when Δv is 10%:

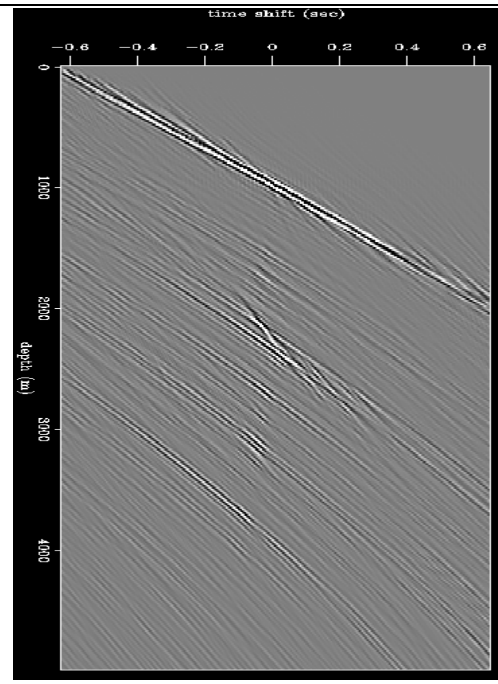

Figure 2: Time-shift gather when migration velocity was too slow below 2500 m depth. The shallow velocity was correct. The energy peaks are shifted away from τ=0, by as much as about -0.15 sec.

Equations (3) and (4) are dip-independent and offset-independent, and extend naturally to v(z) media. A detailed derivation and physical justification of equations (3) and (4) is given in the companion paper (Brown et al., 2008). Equations (3) and (4) represent a horizontal stretch of the time-shift axis of a time-shift gather into units of velocity perturbation, Δv. When the migration velocity is added to the velocity perturbations, the peak energy should represent a measure of the true (RMS) velocity. The depth axis of the gathers is converted to time, and a semblance operator is applied to highlight the energy peaks.
The result is called an “MVFA gather”.

To be clear, MVFA is **not** using a simple traveltime equation to determine velocity on **surface** data—we use the construction to convert a time shift measured **at depth** by prestack depth migration into a velocity perturbation. If the time shift is zero, the velocity perturbation is zero. From Figure 2, we see that the overburden is correct, so the time shift maxima are non-zero only where the velocity is incorrect. MVFA does not make any hyperbolic assumptions about the surface data, or any data for that matter.

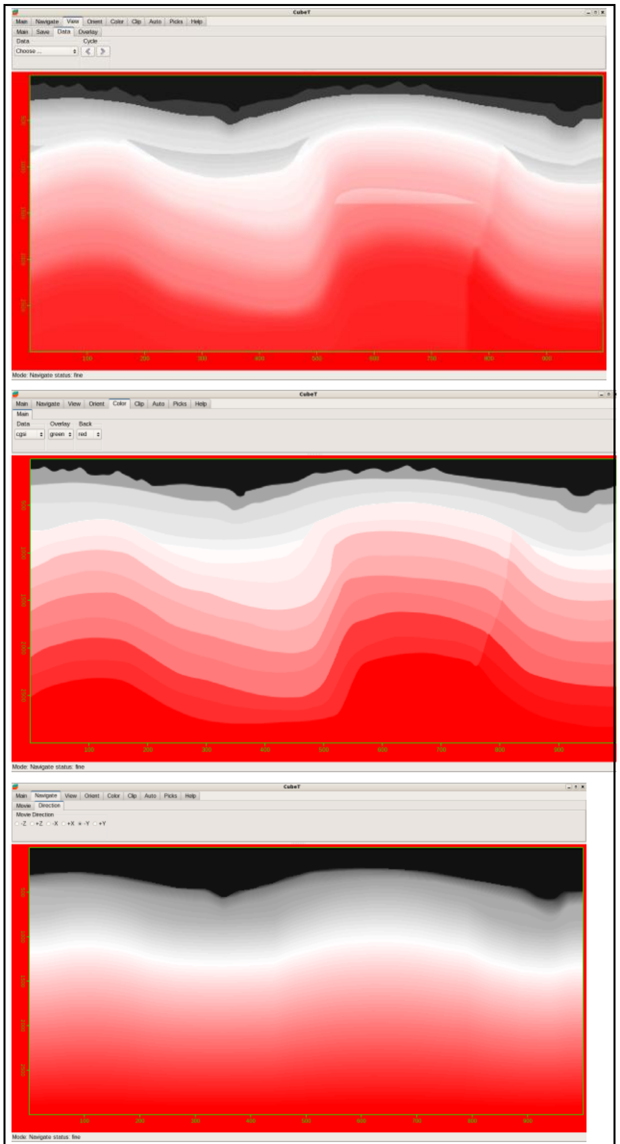

Figure 3: 2D Synthetic Example. Top: velocity model. Center: density model. Bottom: starting velocity for MVFA iterations.

Synthetic example

The top and center panels of Figure 3 shows the earth model used in the synthetic study. The model contains around 200 m of topographic relief, and has a complex velocity profile including lateral velocity variation and velocity inversions, as well as a slow weathering layer of variable thickness. 500 shots were simulated over a 10 km profile, using a pseudo-spectral acoustic wave equation solver, with frequencies up to 45 Hz. Density contrasts produce most of the reflections.

The bottom panel of Figure 3 shows the velocity function used for the beginning of the MVFA iterations. To obtain this model, we “flooded” the subsurface with an assumed weathering layer velocity of 1000 m/s, migrated from topography, and picked the base of the weathering layer. A v(z) model was hung from the base of the weathering layer. The determination of the intercept, v_0, of the v(z) model was accomplished by a migration “sweep” over several v_0 parameters.

Figure 4 compares MVFA gathers computed with the initial velocity, the velocity after two MVFA updates, and the known velocity. Notice how the focusing improves after the MVFA updates, and how the migration velocity more closely matches the semblance peaks, implying convergence. Figure 5 shows zooms of the images corresponding to the velocities in Figure 4. With the initial velocity, the fault is fairly well focused (luck), but the steep dips are obviously not focused. After two MVFA iterations, the fault is well-imaged, as are the steep dips. Some depth errors remain, mostly due to shallow low velocity pods that were not fully inverted for, as we can see from Figure 6. As is common in tomography the estimated model is smoother than the true model. This is a function of our inversion scheme and model parameterization rather than an MVFA limitation, When justified (with a priori information such as well logs or geologic input) discontinuous models can be estimated. Still, comparing the final velocity model Figure 6 to the initial model in Figure 3, we see that two MVFA iterations have done a good job of constructing the large velocity structures.

Conclusions

In this abstract, we introduced a new approach to velocity model building in complex media, Wave Equation Migration Velocity Focusing Analysis, which uses time-shift gathers from wave equation prestack depth migration and converts migration time shifts to perturbations in RMS velocity. We applied two iterations of the MVFA process to a complex land synthetic, and obtain an accurate velocity model, which leads to a well-focused image of the steep dips in the model.

The general concept of using wave equation time-shift gathers has a natural and powerful application to reflection tomography applications, if an angle-dependent time-shift gather can be constructed. Perturbations in velocity (slowness, in fact) are most naturally represented by perturbations in time, which a time-shift gather measures.

Wave Equation Migration Velocity Focusing Analysis

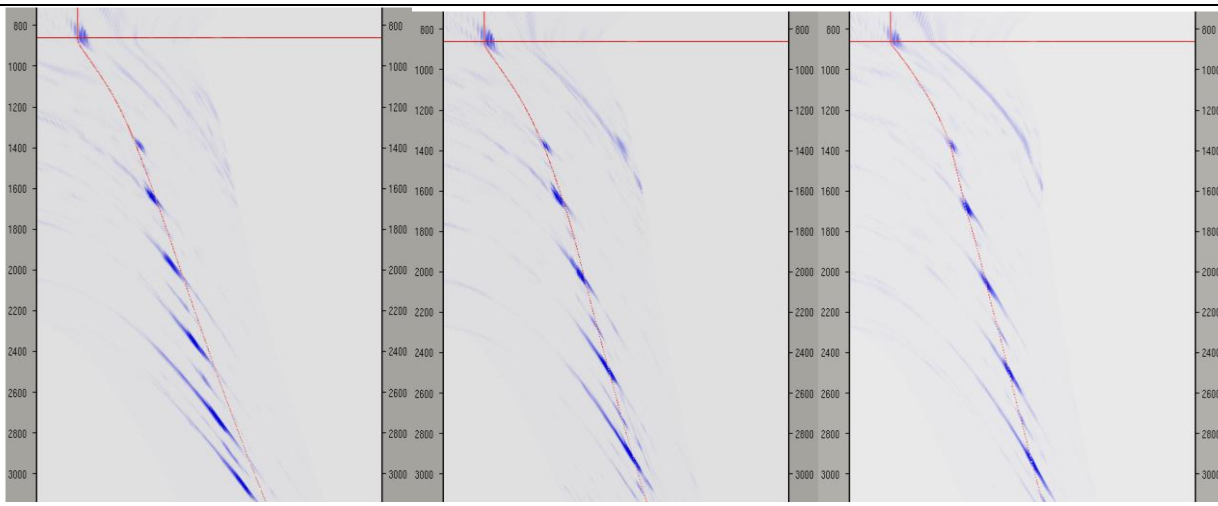

Figure 4: MVFA Panels at x=3200 m (over basin). Vertical axis is time, horizontal axis is RMS velocity (800-2500 m/s). The blue peaks represent velocity implied by analysis. The red line indicates the migration velocity used. Left: After migration with initial velocity. Center: After migration with velocity obtained after two iterations of MVFA updating. Right: After migration with known velocity.

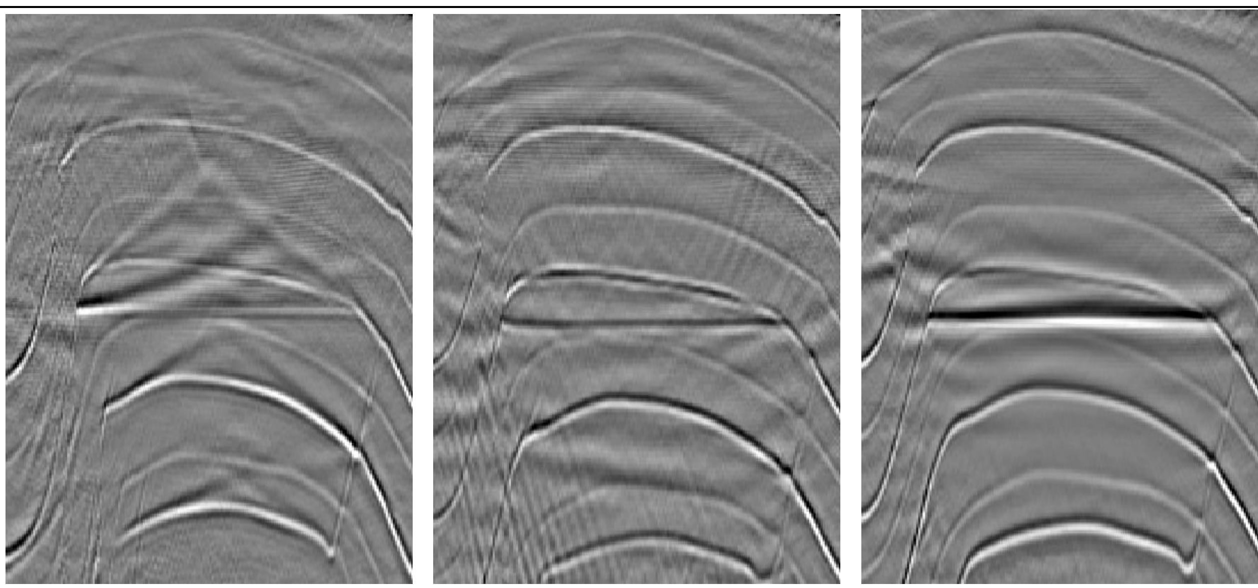

Figure 5: Shot record migration results. Zoomed over anticline for ease of viewing. Left: After migration with initial velocity. Center: After migration with velocity obtained after two iterations of MVFA updating. Right: After migration with known velocity.

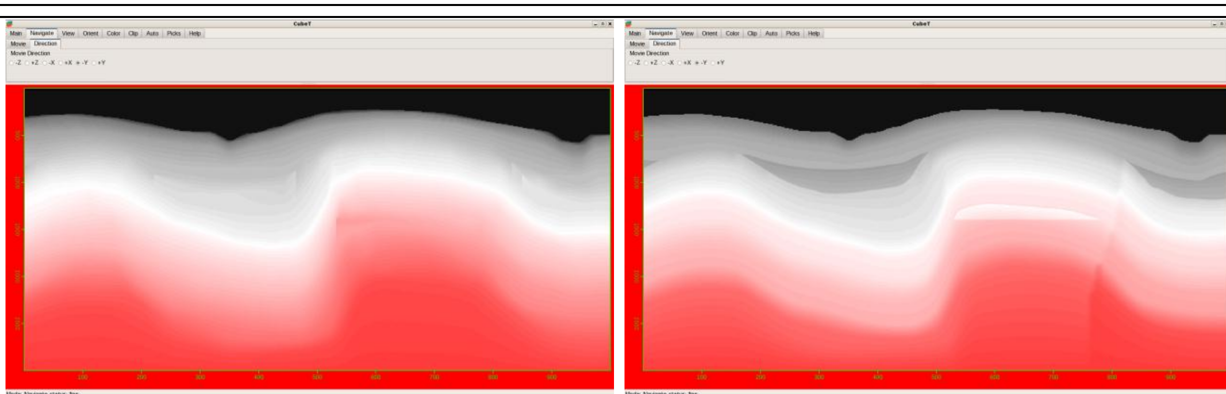

Figure 6: Velocity fields. Left: velocity obtained after two iterations of MVFA updating. Right: known velocity.